Solitary Solutions of the Boiti-Leon-Manna-Pempinelli Equation Using He's Variational Method

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Z. Naturforsch. **63a,** 634 – 636 (2008); received May 5, 2008

A variational formulation is established for the Boiti-Leon-Manna-Pempinelli equation using He's semi-inverse method; three kinds of traveling wave solutions are obtained.

Key words: He's Semi-Inverse Method; Variational Principle; Boiti-Leon-Manna-Pempinelli Equation.
MSC(2000): 34A34, 34C30

1. Introduction

As pointed out by He [1] there generally exist two basic ways to describe a physical problem: 1) differential equations (DE) with boundary or initial conditions; 2) variational principles (VP). The VP model has many advantages over its DE partner: simple and compact in form while comprehensive in content, encompassing implicitly almost all information characterizing the problem under consideration. Variational methods have been, and continue to be, popular tools for nonlinear analysis. When contrasted with other approximate analytical methods, variational methods combine the following two advantages: 1) they provide physical insight into the nature of the solution of the problem; 2) the obtained solutions are the best among all the possible trial functions.

Recently some approximate variational methods, including the approximate energy method (He's variational method) [2 – 7] and the variational iteration method [8 – 25] to soliton solutions, bifurcation, limit cycle, and period solutions of nonlinear equations have attracted much attention. Tatari and Dehghan [16] studied the convergence of the variational iteration method.

He's variational method [26, 27] can be applied not only to weakly nonlinear equations, but also to strongly nonlinear ones. The so obtained results are valid for the whole solution domain [2 – 7].

In [26, 27], Ji-Huan He applied the Ritz method to search for soliton solutions of a nonlinear wave equation (see section 2.2 in [26]). In the present paper we will follow He's basic idea to search for different kinds of solitary solutions.

2. Variational Principle

The Boiti-Leon-Manna-Pempinelli equation reads [28]

$$u_{yt}+u_{xxxy}-3u_{xx}u_y-3u_xu_{xy}=0. \tag{1}$$

In order to seek its traveling wave solution, we introduce a transformation:

$$u(x,y,t)=u(\xi), \tag{2}$$

$$\xi=\lambda t+k_1x+k_2y+\xi_0, \tag{3}$$

where λ, k_1, k_2 and ξ_0 are all arbitrary constants.

Substituting the transformations (2) and (3) into (1) yields

$$k_2\lambda u''+k_1^3k_2u^{(4)}-6k_1^2u''u'=0, \tag{4}$$

where the prime denotes the derivative with respect to ξ. Equation (4) can be simplified as

$$\lambda u''+k_1^3u^{(4)}-6k_1^2u''u'=0. \tag{5}$$

By He's semi-inverse method [29 – 31], we can arrive at the following variational formulations:

$$J_1(u)=\int_0^\infty\left[-\frac{1}{2}\lambda(u')^2+\frac{1}{2}k_1^3(u'')^2+k_1^2(u')^3\right]\mathrm{d}\xi, \tag{6}$$

$$J_2(u)=\int_0^\infty\left[-\frac{1}{2}\lambda(u')^2+\frac{1}{2}k_1^3uu^{(4)}+k_1^2(u')^3\right]\mathrm{d}\xi, \quad (7)$$

$$J_3(u)=\int_0^\infty\left[\frac{1}{2}\lambda uu''+\frac{1}{2}k_1^3(u'')^2+k_1^2(u')^3\right]\mathrm{d}\xi, \quad (8)$$

$$J_4(u)=\int_0^\infty\left[\frac{1}{2}\lambda uu''+\frac{1}{2}k_1^3uu^{(4)}+k_1^2(u')^3\right]\mathrm{d}\xi. \quad (9)$$

3. Solitary Solutions

Using the Ritz method, we can find various solitary solutions. We assume that the solitary solution has the form

$$u_1=A\tanh\xi, \quad (10)$$

where A is an unknown constant to be further determined.

Substituting (10) into (6), we have

$$\begin{aligned}J(u_1)=\int_0^\infty&\Big[-\frac{1}{2}\lambda A^2\mathrm{sech}^4\xi\\&+2A^2k_1^3\mathrm{sech}^4\xi\tanh^2\xi\\&+k_1^2A^3\mathrm{sech}^6\xi\Big]\mathrm{d}\xi\\=-\frac{1}{3}&\lambda A^2+\frac{4}{15}A^2k_1^3+\frac{8}{15}k_1^2A^3.\end{aligned} \quad (11)$$

Making J stationary with respect to A results in

$$\frac{\mathrm{d}J}{\mathrm{d}A}=-\frac{2}{3}\lambda A+\frac{8}{15}Ak_1^3+\frac{8}{5}k_1^2A^2=0 \quad (12)$$

or

$$-5\lambda+4k_1^3+12k_1^2A=0. \quad (13)$$

Solving (13), we obtain

$$A=\frac{5\lambda-4k_1^3}{12k_1^2}. \quad (14)$$

The needed solitary solution reads

$$u_1=\frac{5\lambda-4k_1^3}{12k_1^2}\tanh\xi. \quad (15)$$

We can also assume that the solitary solution has the form

$$u_2=B\mathrm{sech}\xi, \quad (16)$$

where B is an unknown constant to be further determined.

Substituting (16) into (6), we have

$$\begin{aligned}J(u_2)=\int_0^\infty&\Big[-\frac{1}{2}\lambda B^2\tanh^2\xi\,\mathrm{sech}^2\xi\\&+\frac{1}{2}B^2k_1^3\mathrm{sech}^2\xi\\&+k_1^2B^3\mathrm{sech}^3\xi\tanh^3\xi\Big]\mathrm{d}\xi\\&=-\frac{1}{6}\lambda B^2+\frac{1}{2}B^2k_1^3-\frac{2}{15}k_1^2B^3.\end{aligned} \quad (17)$$

Making J stationary with respect to B results in

$$\frac{\mathrm{d}J}{\mathrm{d}B}=-\frac{1}{3}\lambda B+Bk_1^3-\frac{2}{5}k_1^2B^2=0. \quad (18)$$

So, it is not difficult to get B from (18), which reads

$$B=\frac{15k_1^3-5\lambda}{6k_1^2}. \quad (19)$$

We, therefore, obtain the following solitary solution:

$$u_2=\frac{15k_1^3-5\lambda}{6k_1^2}\mathrm{sech}\xi. \quad (20)$$

Now we consider another case:

$$u_3=C\mathrm{sech}\xi\tanh^2\xi. \quad (21)$$

By the same operation as illustrated above, we obtain

$$C=\frac{11(1199k_1^3-213\lambda)}{4368k_1^2}, \quad (22)$$

$$u_3=\frac{11(1199k_1^3-213\lambda)}{4368k_1^2}\mathrm{sech}\xi\tanh^2\xi. \quad (23)$$

4. Conclusion

We established variational formulations for the concerned problem by He's semi-inverse method, and various kinds of solitary solutions can be derived using the Ritz method without any difficulty.

Acknowledgements

This work was supported by Nanjing University of Information Science and Technology (Y626).

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